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**Seek-Time Improvement
in a Random-Access File by
Application of an Adaptive Element**

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by
W. S. Buslik*

December 1963

Technical Report No. 6762-1

Prepared under
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Jointly supported by the U.S. Army Signal Corps, the
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1964

*Author sponsored by an IBM Fellowship

SYSTEMS THEORY LABORATORY

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SEEK-TIME IMPROVEMENT IN A RANDOM-ACCESS FILE
BY APPLICATION OF AN ADAPTIVE ELEMENT

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W. S. Buslik*

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Systems Theory Laboratory
Stanford Electronics Laboratories
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ABSTRACT

An Adaline (adaptive linear neuron) can be trained to distinguish between sets of inputs. In general, the quantized output is used. This report investigates the usefulness of the analog output of Adaline for measuring the frequency of occurrence of a number of different events. Each event is more or less arbitrarily associated with a pattern and it is shown that the degree to which Adaline has been trained to recognize any one of these patterns can be used as a measure of the frequency of occurrence of the associated event. The application of this use of Adaline to a random-access file is simulated in order to show its use in reducing average access time.

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SYMBOLS

$A(a_1, a_2, a_3, a_4, \dots, a_N)$	pattern vector
A_i	gain setting on input i to Adaline
C	time constant factor
C_i	confidence level (analog output) of an Adaline when interrogated with pattern P_i
$MM_E(N)$	mean magnitude of cross-effect factor for N-dimensional case for $N = \text{even}$
$MM_O(N)$	mean magnitude of cross-effect factor for N-dimensional case for $N = \text{odd}$
N	number of inputs to Adaline
N_O	total number of different events
P_i	pattern assigned to event i
Pr_i	"probability" of occurrence of event i as sensed by Adaline
$Prob_i$	probability of occurrence of event i or access to record i
S_i	access or seek time to record i
$Y(y_1, y_2, \dots, y_N)$	pattern vector

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Appreciation is also expressed for the opportunity to work under Dr. Bernard Widrow's guidance during the course of this research.

I. INTRODUCTION

The problem to which this report is devoted is believed to occur frequently in different forms and will first be described in general terms and then as applied to a specific example, a random-access memory file. The schematic diagram of Fig. 1 shows this general case. A finite number of identical events take place repeatedly in a given total number of events. The number of possible different events is N_0 . The probabilities of events 1 through N_0 occurring are designated by Prob_1 through Prob_{N_0} respectively. The total number of events is assumed to be 100. These probabilities are not known initially, however, and are to be determined such that the resulting probabilities Pr_1 through Pr_{10} represent a recent measurement. In other words, it is desired to forget the distant past in favor of more emphasis on the recent past. It is believed that Adaline (adaptive linear neuron) [Ref. 1] can solve this task in the most natural manner. Before going into "how," it is useful to consider the random-access memory application.

As a result of the relative slowness of mechanical mechanisms that perform one or more of the accessing motions, the access or seek time in a random-access file depends greatly on the location in the file of the record to which an access is to be made. The simplest case is that in which the mechanism returns to a home position after each access, such that the records nearest the home position have the shortest seek time, and those farthest away, the longest. It is quite unusual for all records to have more or less equal activities, i.e., equal number of accesses in a given period of time. In fact, the usual situation is that in which a large fraction of the total number of accesses is made to a small fraction of the total number of records. For instance, 80 percent of accesses may be made to 20 percent of the records. In such a situation it would obviously be advantageous to place these most active records near the home position for the shortest possible average seek time. In order to permit occasional rearranging of the records in the file to obtain this advantage, it is necessary to determine the probabilities of seeks to all the records of the file. Figure 2 shows this file-access model in

EVENT NO: 1 2 3 4 5 ... N_0

TAKES PLACE WITH THESE PROBABILITIES: Prob_1 Prob_2 Prob_3 ... Prob_{N_0}

PROBLEM: HOW TO OBTAIN THE RELATIVE MAGNITUDES OF THESE Prob'S WITH ADALINE

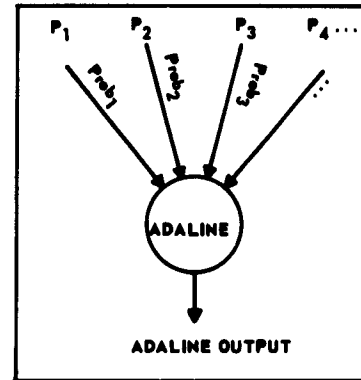
PROCEDURE:

ONE PATTERN P_1 P_2 P_3 P_4 P_5 ... P_{N_0}
IS ASSIGNED TO EACH EVENT

EACH TIME EVENT 1 TAKES
PLACE, ADALINE IS ADAPTED
TO PATTERN P_1

ON INTERROGATION, CONFIDENCE
LEVELS C_1 C_2 C_3 ... ARE
OBTAINED

$$\text{ADALINE OUTPUT} = \frac{1}{1 - |C_1|} = \text{Pr}_1$$



CONTENTION: THE Pr_1 ARE A MEASURE OF THE Prob_1 , AT LEAST TO THE
EXTENT THAT IF

$$\text{Prob}_a > \text{Prob}_\beta > \text{Prob}_\gamma > \dots$$

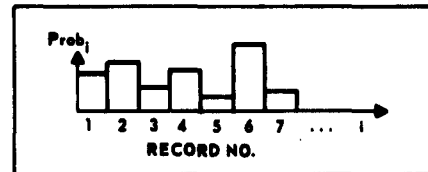
THEN

$$\text{Pr}_a > \text{Pr}_\beta > \text{Pr}_\gamma > \dots$$

FIG. 1. GENERAL PROBLEM.

RECORD NO.: 1 2 3 4 5 6 7 ... N_0

PROBABILITY OF ACCESS TO ABOVE RECORDS: $Prob_1$ $Prob_2$... $Prob_{N_0}$

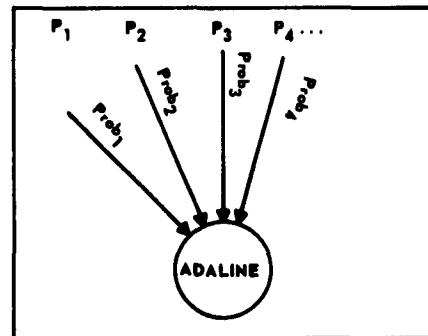


PROCEDURE:

ASSIGN PATTERNS (P_1 P_2 P_3 P_4 ...)

TRAIN ADALINE WITH THE P_i IN
NUMBERS PROPORTIONAL TO THE
 $Prob_i$

INTERROGATE ADALINE WITH:



PATTERNS—

P_1 P_2 P_3 P_4 P_5 ...

CONFIDENCE LEVELS—

C_1 C_2 C_3 C_4 C_5 ...

$$ADALINE OUTPUT = \frac{1}{1 - |C_1|} = Pr_1 \quad Pr_2 \quad Pr_3 \quad Pr_4 \quad Pr_5 \quad \dots$$

SORT THE Pr IN DESCENDING ORDER OF MAGNITUDE: $Pr_6 \geq Pr_2 \geq Pr_4 \geq Pr_1$
... Pr_j ...

SEEK TIMES FOR POSITIONS 1 TO N_0 : S_1 S_2 S_3 S_4 ... S_1 ...

FORM PRODUCTS: $S_1 \times Prob_6$, $S_2 \times Prob_2$, $S_3 \times Prob_4$... $S_1 \times Prob_j$...

$$AVERAGE SEEK TIME = \frac{1}{N} \sum_{i=1}^{i=N_0} (S_i \cdot Prob_j)$$

FIG. 2. MODEL OF FILE.

schematic form. The first line of Fig. 2 simply shows the record numbers, later to be limited to 10. Next, Prob_i is the actual probability of an access to the i^{th} record. In order to bring Adaline into the picture, a pattern is assigned to or generated from each record and this pattern is compatible in form and dimension with the number of inputs to Adaline. (A brief review of the operation and function of Adaline is contained in the next chapter.)

In the experiments that follow, the number of inputs to Adaline are taken as 32 and the signal levels as +1 or -1. Whenever an access is made to one of the records in the file, the corresponding assigned pattern is presented to Adaline and an adaption is performed. Thus Adaline is trained to recognize these patterns more or less reliably depending on how many accesses to this record (pattern) have been made recently. The term "recently" is used because Adaline has the ability to forget gradually those patterns that are inactive or of low activity. Whenever an opportunity arises, because of a break in the activity of the file as a whole, the record probabilities are determined by interrogating Adaline with all the patterns that have been assigned to records. Only the analog output is required and the probability of access is presumed to be measured by the reciprocal of $1 - |\text{confidence level}|$. Since the confidence level is close to 1 after many adaptations, this measure seems reasonable. The records are then sorted into a sequence having descending probabilities of access. The record with the highest probability is thus located nearest the home position; and the lowest, farthest away. This process is repeated periodically to keep the file up to date with respect to changes in the probability function. Provided enough time is available to rearrange the records more or less continually, the file may be maintained at or near optimum average seek time. (This rearrangement may be performed by having one or more spare tracks on which the records would be rewritten before placing them in their new locations.)

It is evident that the more nearly the order of the probabilities Pr_i corresponds to the order of the actual probabilities Prob_i , the more effectively will the average seek time approach the best-obtainable average seek time. How well this goal can be reached is investigated

here in a number of experiments with simulated Adalines and simulated file operations.

II. ADALINE OPERATION

This chapter presents a brief review of the elements making up an Adaline and their operation insofar as they are relevant to the specific application of the Adaline considered here [Refs. 1, 2]. For this purpose Adaline may be represented by Fig. 3. There are N inputs to Adaline. These may assume the values $+1$ or -1 only. Each input is amplified with a gain of A_i which is continuously adjustable under the control of the adaption processor, or adapter. The sum of all the amplified inputs is formed, which is the "confidence level" for the particular input pattern. The confidence level, as well as the inputs themselves, are fed into the adapter. From this information the adapter determines what changes to make to the gains. This decision making forms the adaption procedure.

The principal adaption procedure used here is the following: a "desired output" is formed which has the magnitude 1 and the same sign as the confidence level formed before the adaption. This procedure is considered a "bootstrap adaption" since no a priori desired output exists. The error is the difference between the just-formed desired output and the confidence level. It may be positive or negative. Adaption then consists of changing all the gains by an amount proportional to the product of the error and the input to that gain. The constant of proportionality is $1/32C$, in which C is a time-constant factor. Thus, the larger the C the more adaptations will be required to reach a confidence level close to 1 in magnitude. With $C = 1$ the confidence level of 1 would be obtained after only one adaption. Here, values for C of 6, 10, and 12 have been used.

As already mentioned, the fraction $1/(1 - |\text{confidence level}|)$ is considered to be a measure of the probability of an access to the record to which the pattern corresponds. This fraction can be obtained in an interrogation which is simply the first part of the adaption, viz., the summing of the amplified inputs yielding the confidence level.

A different adaption procedure is used only briefly and consists of adding a fixed increment to each gain equal to the product of the input and the sign of the confidence level. This increment may be multiplied

by a reduction factor to keep the size of the confidence level within reason, but its size has no other effect. The confidence level is now used as a measure of probability of access to that record. To avoid misunderstanding, it should be mentioned that only the experiment resulting in Fig. 12 (Chapter V) uses this latter adaption procedure and probability measure.

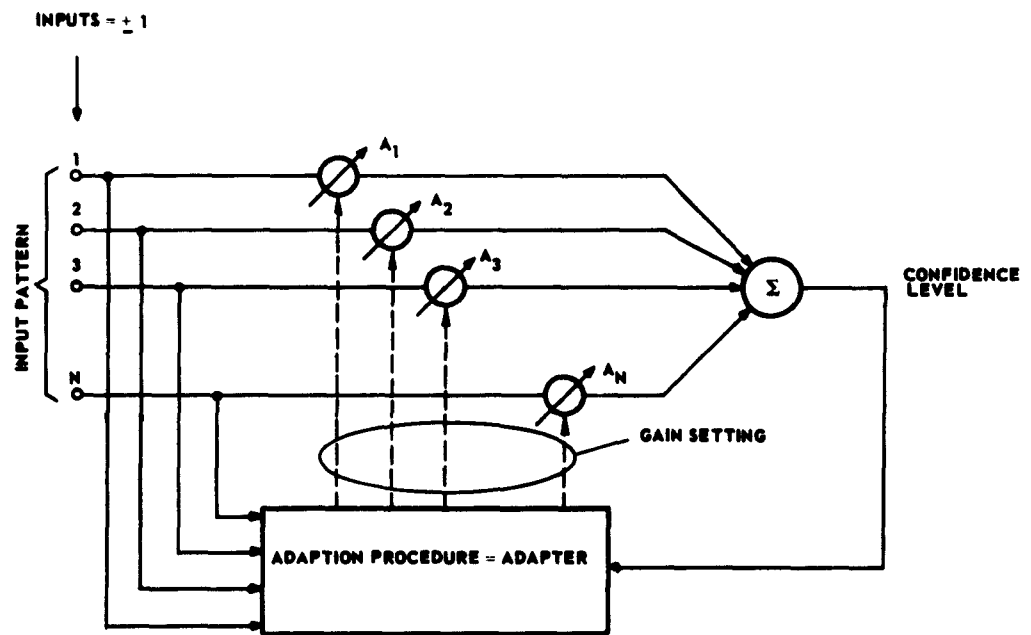


FIG. 3. SCHEMATIC OF ADALINE BASED ON REFS. 1 AND 2.

III. METHOD AND PREVIEW OF EXPERIMENTS

All experiments were performed by simulation on the IBM 7090 computer, assuming linear amplifiers for all inputs. Actually, one simulation was made assuming that the amplifiers would saturate like an arctangent function. The result was not essentially different from the same simulation assuming linear amplifiers. The program is very time-consuming and was not considered worth pursuing.

To begin with, a certain probability-of-access function was assumed. The probabilities were assumed to only two significant figures so that they could be reproduced exactly in 100 accesses. For the complete file simulation the probability functions were restricted further as described under that experiment (Chapter VI). But generally, they were simple functions often containing pairs of records with equal probabilities to show whether these were sensed as equal by Adaline. After the number of accesses (out of 100) to each record is determined, the sequence is shuffled as follows:

1. Batch the accesses in ascending order by size, smallest batch first, to form the first sequence. (All accesses to the same record form a "batch.")
2. Use the top record from the first sequence (for convenience, designated as A) to form the beginning of a second sequence.
3. Pick the next record (B) from the first sequence and place it on top of the second sequence, then switch the bottom record so that it becomes the top record.
4. Transfer the next record (C) from the first sequence to the top of the second sequence, and again switch the bottom record to the top.
5. Continue alternating records from the first sequence with the bottom record of the second sequence until there are no records left. In effect, the new sequence now becomes the old.
6. Repeat steps 1 through 5 three times.

This process results in a fairly random looking sequence, yet it can be programmed and repeated for any probability function. Every experiment, unless otherwise stated, was begun with all gains reset to 0. This resetting at least insured repeatability of any experiment, although it does not represent a realistic situation. The effect of the resetting of

the gains is, however, believed to be slight after a number of adaptions. This fact is particularly true since at least 200, and often several thousand, adaptions were performed on every experiment.

Most experiments were made with random and mutually orthogonal sets of input patterns. (Patterns are orthogonal if their scalar products vanish.) With random patterns the indicated probabilities (or output) are dependent on the selection of the pattern set and on how the patterns are assigned to the various records. To demonstrate this effect on the one hand and to avoid it (by averaging) on the other, pattern assignments were rotated. Rotation of pattern assignment is simply advancing every pattern so that it is now assigned to the succeeding record. The last pattern thus becomes assigned to the first record. After N_0 rotations, where N_0 is the number of different records and patterns, the original assignments are restored. In most of the experiments, complete assignment rotation was performed.

In order to describe Adaline's accuracy in detecting probabilities in these experiments, the given (i.e., the actual) probabilities may be compared with the output of Adaline. This comparison can be in the form of an actual plot of the probability function after every 5, 10, or 25 adaptions and comparing these graphs with the graph of the given function. These graphs are shown on the left side of Fig. 4. It is, however, very tedious to plot these functions and it is often difficult to compare them when there are many or if they cannot be shown adjacent to one another. The correlation coefficient between the given and the output probability functions is as good as any single figure of merit. It is shown alongside the probability functions in Fig. 4. (These graphs were taken from the first 30 adaptions on 10 random patterns.) To get a better feel for the significance of the correlation coefficient, a number of fictitious output functions are plotted in Fig. 5 with a given-probability-function graph. The correlation coefficients range from +0.994 to -1.000 in this case. Most of the more complicated experiments are described by the associated correlation coefficients only.

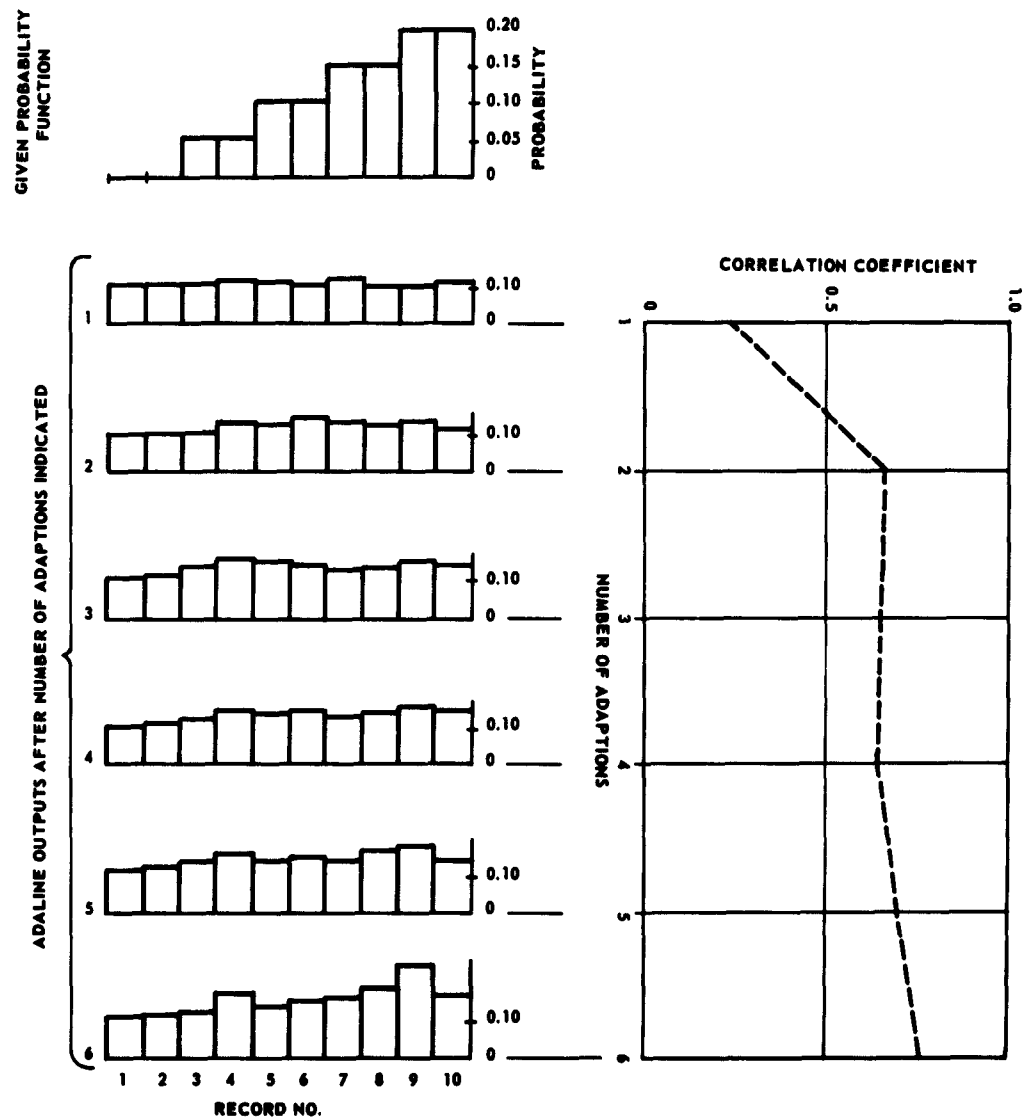


FIG. 4. CORRELATION COEFFICIENTS AND PROBABILITY FUNCTIONS.
(10 random patterns.)

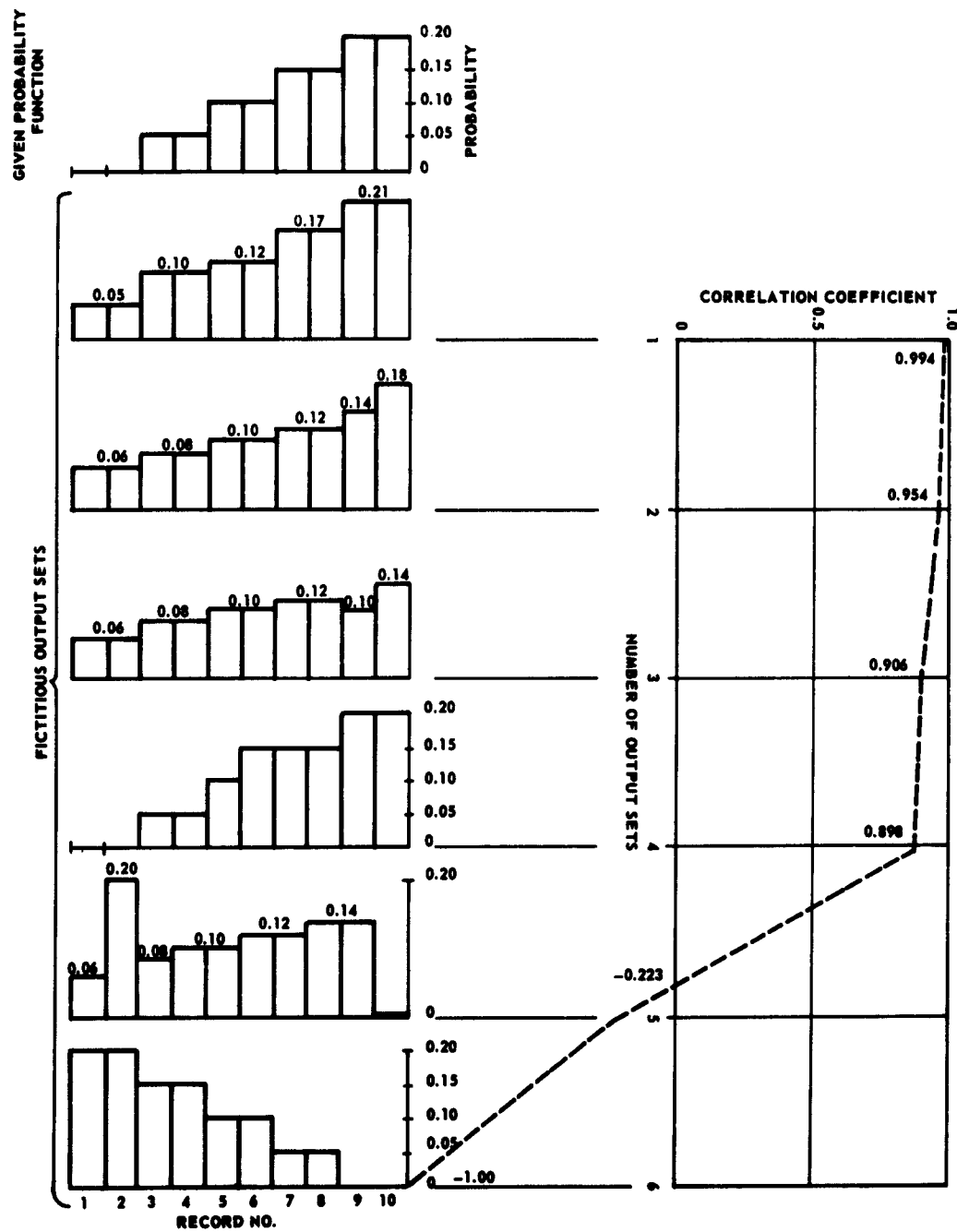


FIG. 5. CORRELATION COEFFICIENTS AND FICTITIOUS ADALINE OUTPUTS.

Basically, three types of experiments were performed: static and dynamic adaption and file simulation. An attempt is made to describe the behavior of the Adaline output with regard to the following:

1. Differences between orthogonal and random patterns.
2. Effect of the time-constant factor.
3. Effect of the number of patterns for a given number of Adaline inputs.
4. Effect of the size of the discrepancy between adjacent record probabilities on the accuracy of the output.

In the dynamic experiments the following additional information was sought:

5. Effect of change in probability function on recovery.
6. Behavior under repeated changes of probability function.

File simulation consisted of just one experiment that included many of the above aspects, plus an overall effectiveness measure as far as seek-time improvement is concerned.

IV. STATIC BEHAVIOR

Figures 6 and 7 show the input probability functions and output functions with orthogonal and random patterns respectively. Since the given probability function is truly represented only after 100 records or patterns have been adapted, one would not expect the input function to be reproduced by the output with any fidelity until after 100 adaptations. As is evident from the graphs, the output after 100 adaptations in the case of the orthogonal patterns is a faithful reproduction of the given function. In the random case there is evidence of gross error in pattern 4. This effect is due to the confidence level for that pattern being influenced by one other pattern or a combination of several other patterns. This type of "cross adaption" will be discussed in more detail in Chapter VII. It will be shown that the correction vector, which is added to the gain vector in an adaption, is parallel to the pattern vector on which the model is adapting. The confidence level is the scalar product of the gain vector and the pattern vector. Thus it is evident that adaption on one pattern will not affect the confidence level of another pattern that is orthogonal to the first one. One therefore would expect the "exact" reproduction of the input probabilities as shown in the bottom graph of Fig. 6.

Figure 8 summarizes the results of seven experiments investigating effects of gross differences in the probabilities of the input probability function. To start with, record 10 has a probability of 0.79, and record 9 has probability 0.06, and each successively lower record number has a probability 0.01 less than the preceding one. The 10 patterns are random patterns and pattern assignments were rotated completely so that each pattern was assigned to each record once. Output probabilities were determined by interrogation of Adaline after 100 and 200 adaptations. This procedure then resulted in 20 sets of output probabilities. These sets were averaged, with the results as shown in Fig. 8.

In the succeeding experiments the probability of record 10 was reduced in steps of 0.09, and the remaining probabilities all increased by 0.01. The patterns remained the same throughout. The fidelity of the output remained excellent to the last experiment in which the two

largest input probabilities were 0.25 and 0.12. This fidelity is an indication of successful operation where relatively large discrepancies in input probabilities exist, which after all are the situations of greatest interest because only in these cases can some improvement in access time be expected.

Summarizing, static behavior demonstrates Adaline's ability to reproduce exactly the input probabilities in the case of orthogonal patterns and nearly so, on the average, for random patterns.

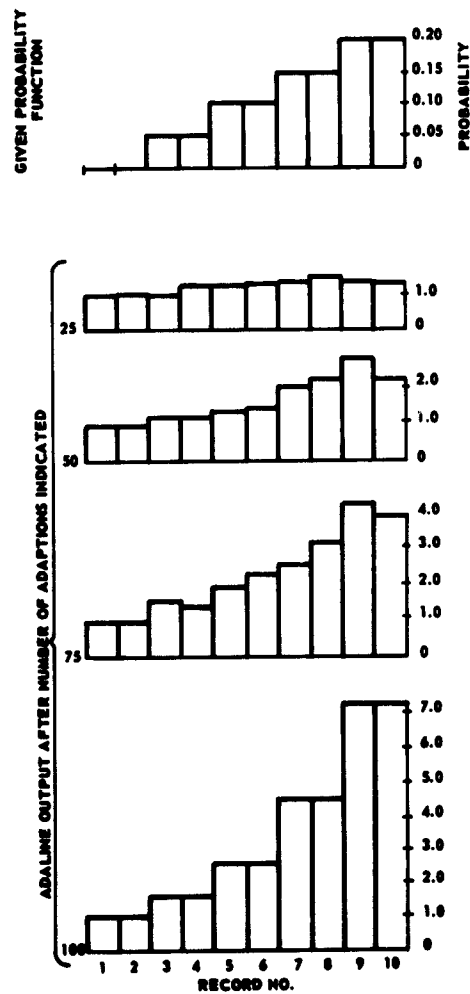


FIG. 6. ADALINE ADAPTION WITH ORTHOGONAL PATTERNS.

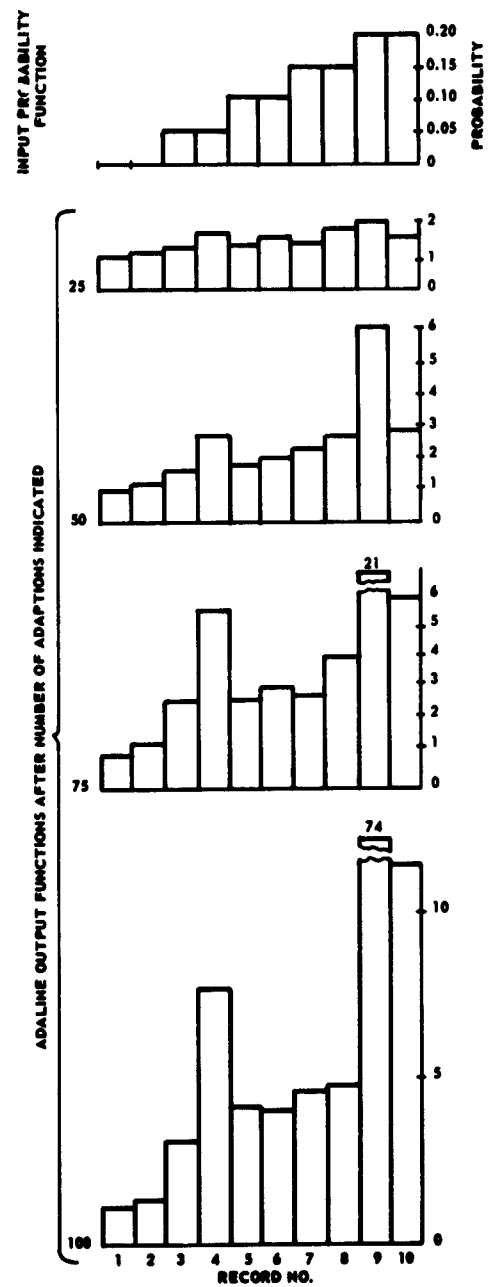


FIG. 7. ADALINE ADAPTION WITH RANDOM PATTERNS.

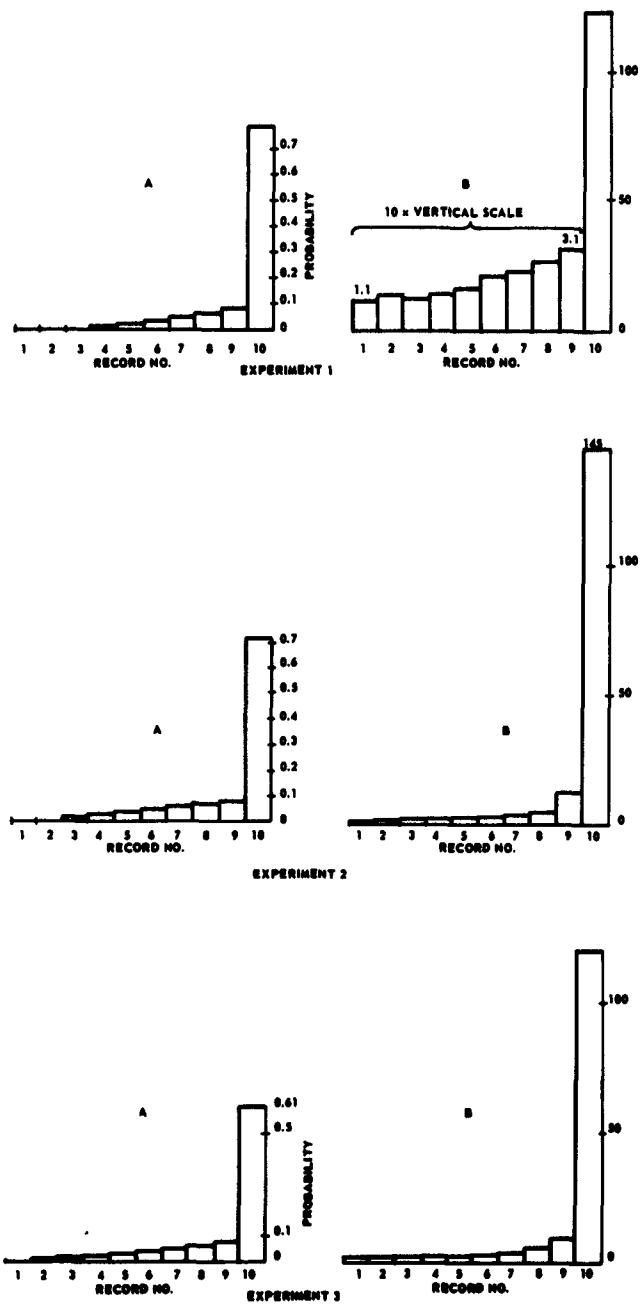


FIG. 8. ADALINE ADAPPTIONS ON FUNCTIONS WITH GROSS DIFFERENCES BETWEEN PROBABILITIES.
(A = given probability function; B = average Adaline output.)

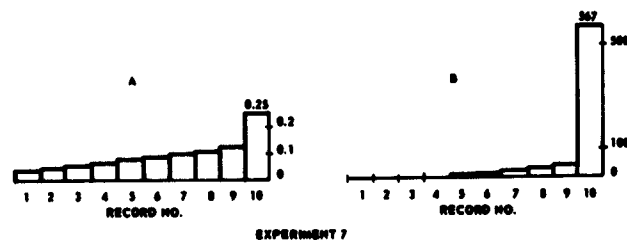
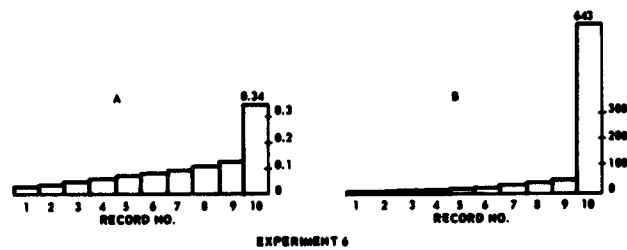
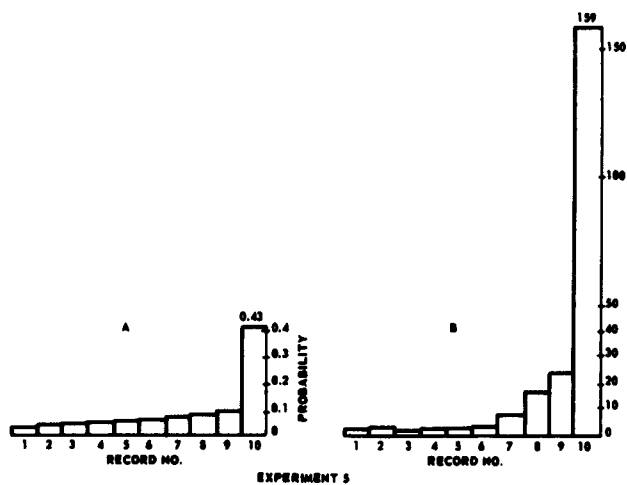
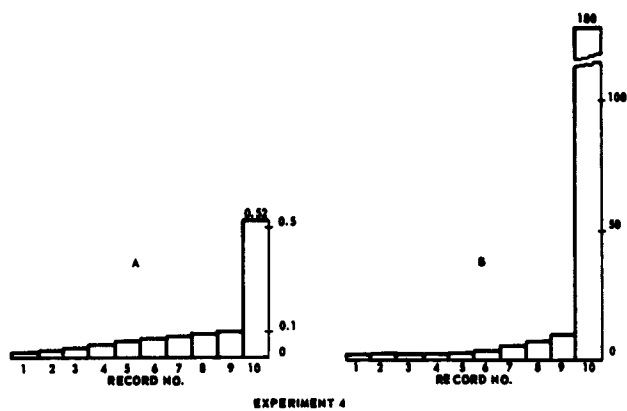


FIG. 8. (Continued)

V. DYNAMIC BEHAVIOR

The first dynamic experiment was an adaption to a flat probability function after Adaline had already been adapted 1000 times to the probability function shown in the first diagram of Fig. 9. The output probability function is shown directly below. The output functions resulting after the change are reproduced for every 100 adaptations. This experiment was done with orthogonal patterns and must be considered one of the more difficult adaptations because of the absolutely flat input. Adaption was slow, but after 1100 adaptations the output approached a flat function. The logarithmic scale for the 12 output diagrams is the same but was not converted to "probability" dimension, i.e., the sum of the ordinates is not 1.

The remaining dynamic experiments are described by changes in correlation coefficient between the input function and the output. It would then be expected that a meaningful measure could be obtained only every 100 adaptations, because only then are the input probabilities coincident with the actual pattern frequencies. In the case of simple input functions, actual frequency distributions resembling the input functions are reached after only a dozen or so adaptations because of the effective scrambling of record sequences. It was therefore considered worthwhile in some experiments to show the correlation coefficient for every ten adaptations. The six plots of Fig. 10 each relate to behavior after a single change in input probability function to the inverse functions.

In Figs. 10a and 10b, ten random patterns were used and the difference between the two figures is only the time-constant factor, which is 12 for Fig. 10a and 6 for Fig. 10b. Adaption to the new input is not too good in either case, probably because of an unfortunate choice of the pattern set. For random patterns good reproduction of the input can be expected only in a statistical sense and not necessarily for every pattern set in a single experiment. Similar comments are in order for a comparison of Figs. 10b and 10c, which differ only in the number of patterns employed. It is impossible, however, to produce exactly the

same input probability function for 24 patterns as for 10. The 24 patterns selected include the 10 used in the preceding experiment. It is evident that adaption to both the original input function as well as to the inverse function is slower for the larger number of patterns, as would be expected.

The next three experiments are a repetition of the preceding three except that they were performed with orthogonal patterns. Figures 10d and 10e show the effect of change in time-constant factor. The effect is small and it would appear that a time-constant factor of 12 gives a somewhat better correlation coefficient. Since in an actual application the time-constant factor would have to be chosen for the particular dynamic conditions prevailing, in this case 12 would be the preferred choice. It is interesting that in both experiments the correlation coefficient shows a very sudden change from a large negative value to a large positive value exactly 100 adaptions after the change in input functions. This sudden change indicates that adaption to the existing frequency distribution is fairly complete at some time prior to the 100th adaption after the change, and when the existing frequency distribution suddenly coincides with the given input-function, correlation becomes very good suddenly. This argument also explains the fairly rapid change but not the step at the 200th adaption. Examination of the output at that point reveals that it is a completely flat function, unfortunately producing a large correlation coefficient with the input function, which is approximately a straight line. This finding points up one of the shortcomings of the correlation-coefficient presentation.

Figures 10e and 10f again show the effect of an increase in the number of patterns, which results in definitely slower adaptions. The step has disappeared because of the incomplete adaption near the 200th adaption. Again comparisons of these two experiments should not be carried too far because of the differences introduced by the change in the input function and the addition of new patterns to make up 24. This latter effect would not be as drastic as in the case of random patterns, however, because orthogonality cross effects are not present in adaption.

PROBABILITY FUNCTION IS CHANGED FROM NO. 1: 0.00 0.00 0.05 0.05 0.10 0.10 0.15 0.15 0.20 0.20
 TO NO. 2: 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10

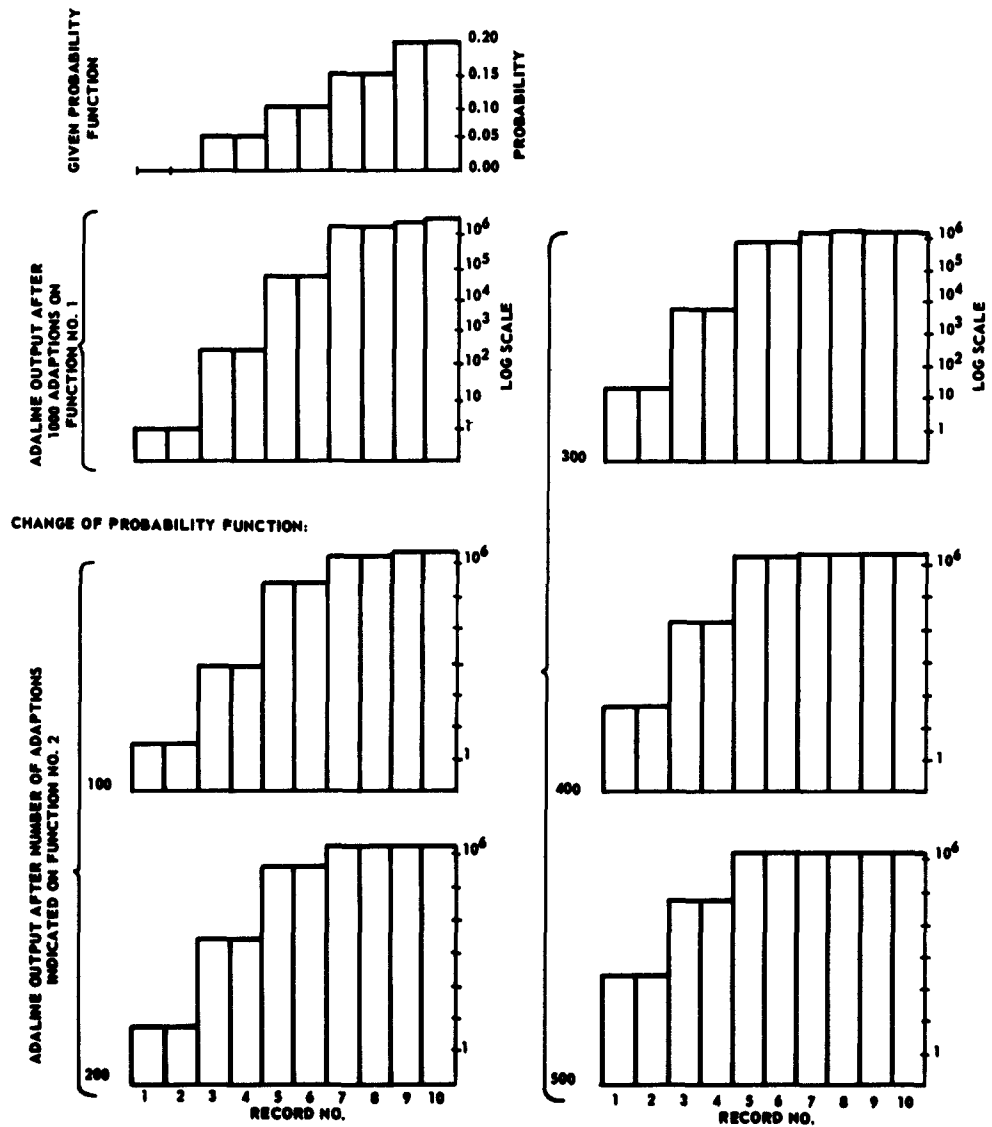


FIG. 9. DYNAMIC BEHAVIOR WITH A SINGLE CHANGE IN PROBABILITY FUNCTION.

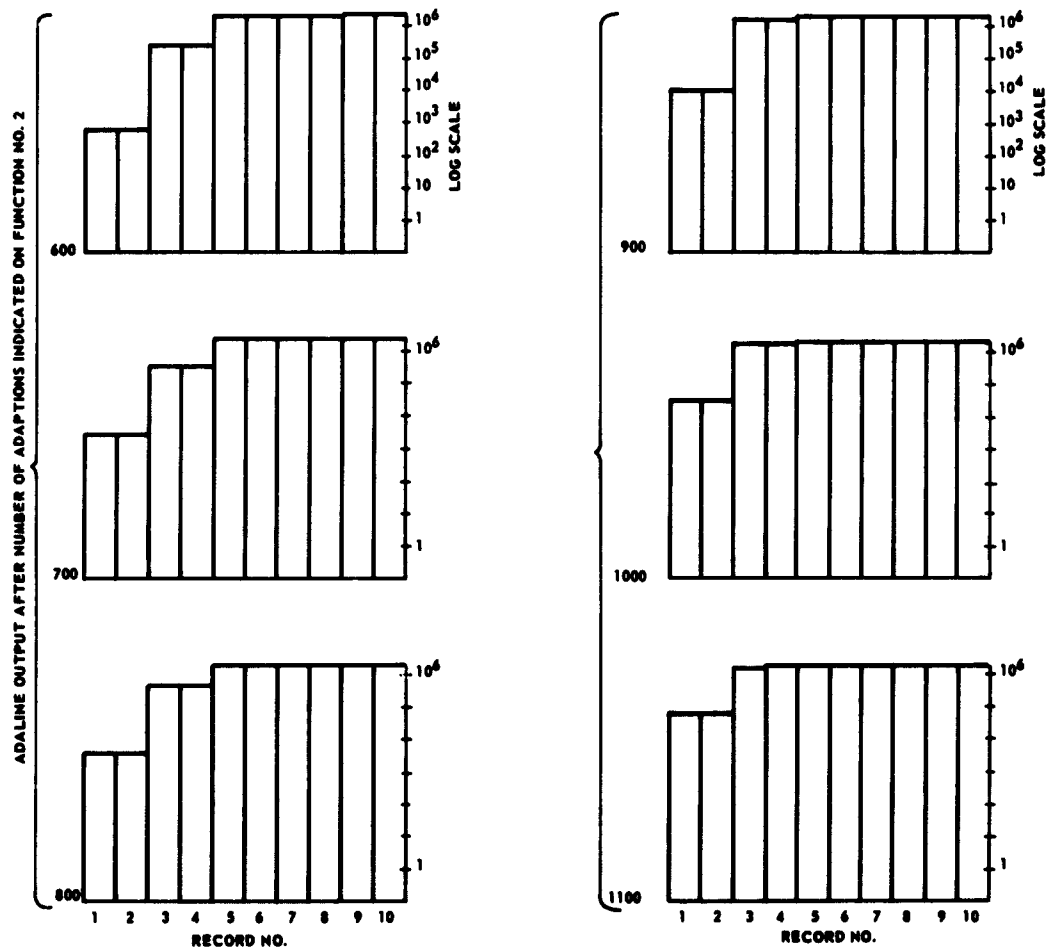
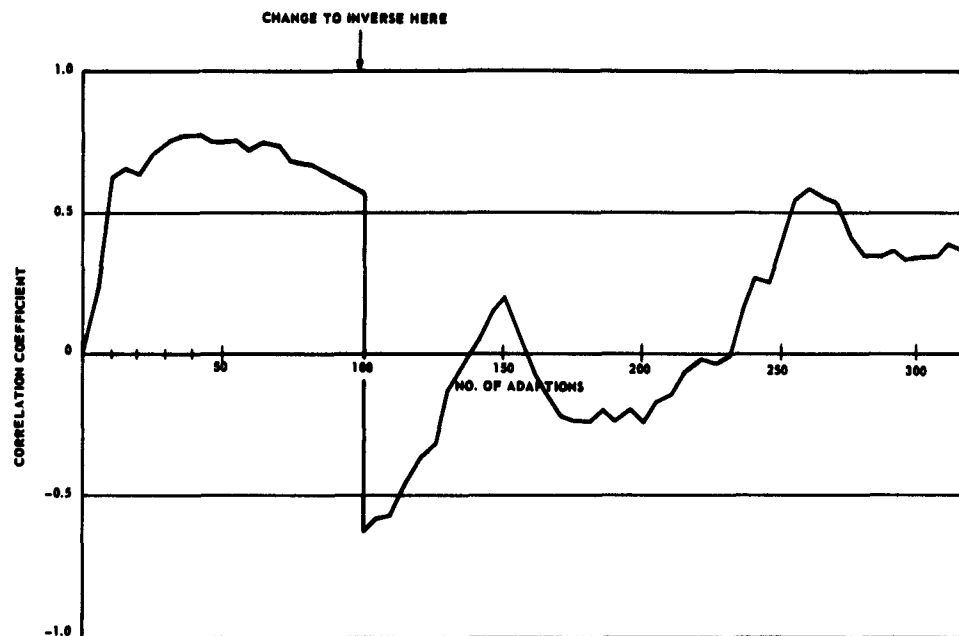
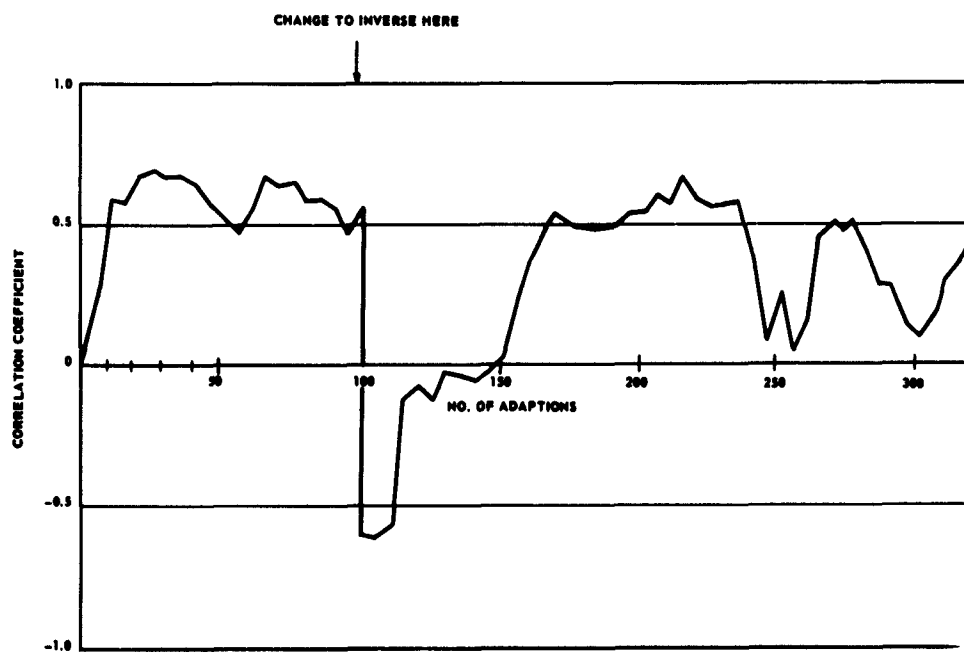


FIG. 9. (Continued)

PROBABILITY FUNCTION: 0.00 0.00 0.05 0.05 0.10 0.10 0.15 0.15 0.20 0.20
 IS CHANGED TO "INVERSE": 0.20 0.20 0.15 0.15 0.10 0.10 0.05 0.05 0.00 0.00

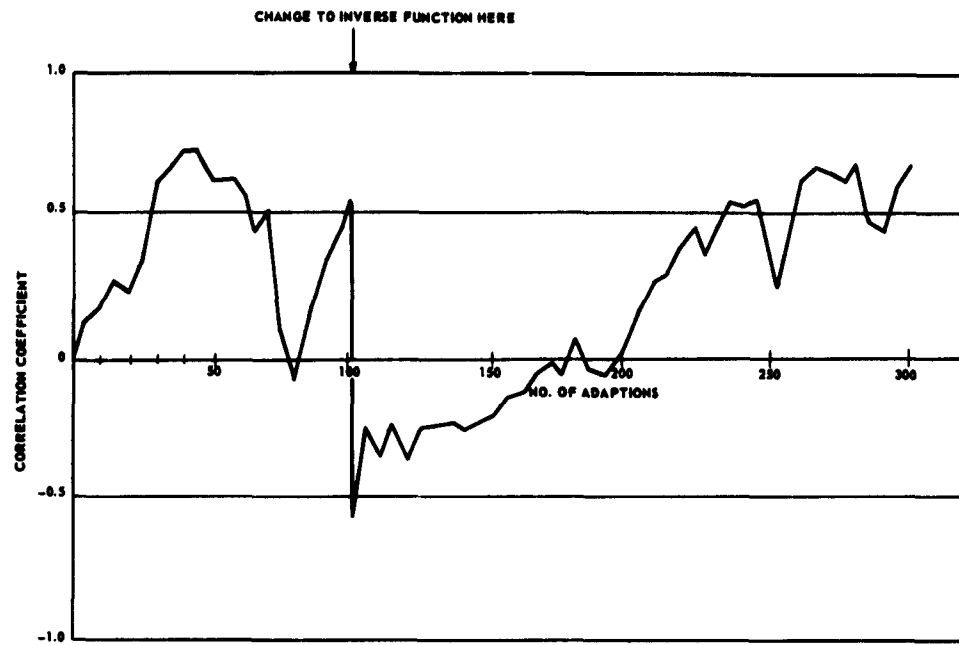


a. 10 random patterns, $C = 12$

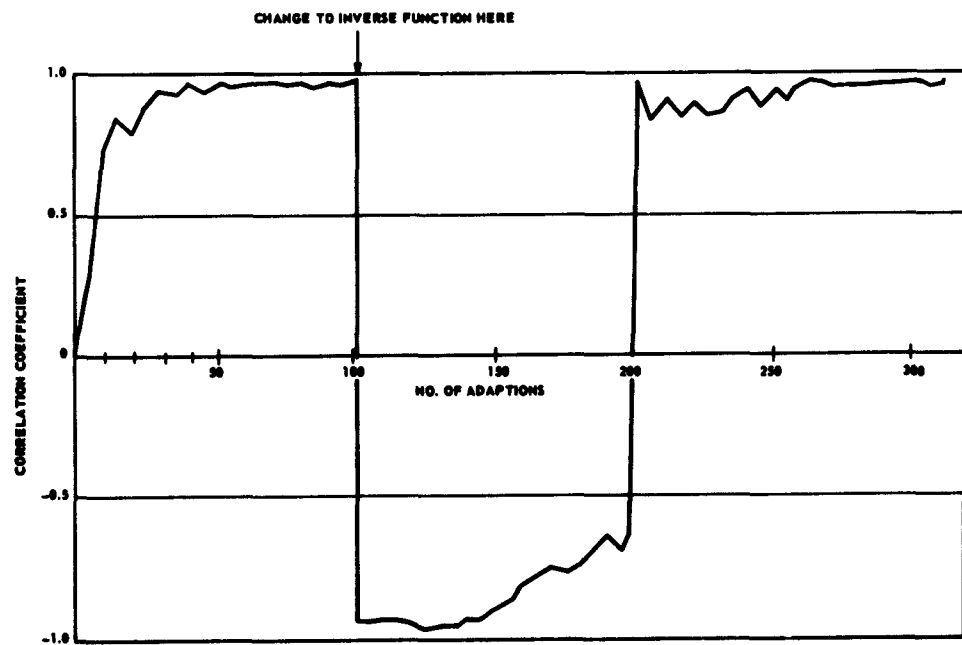


b. 10 random patterns, $C = 6$

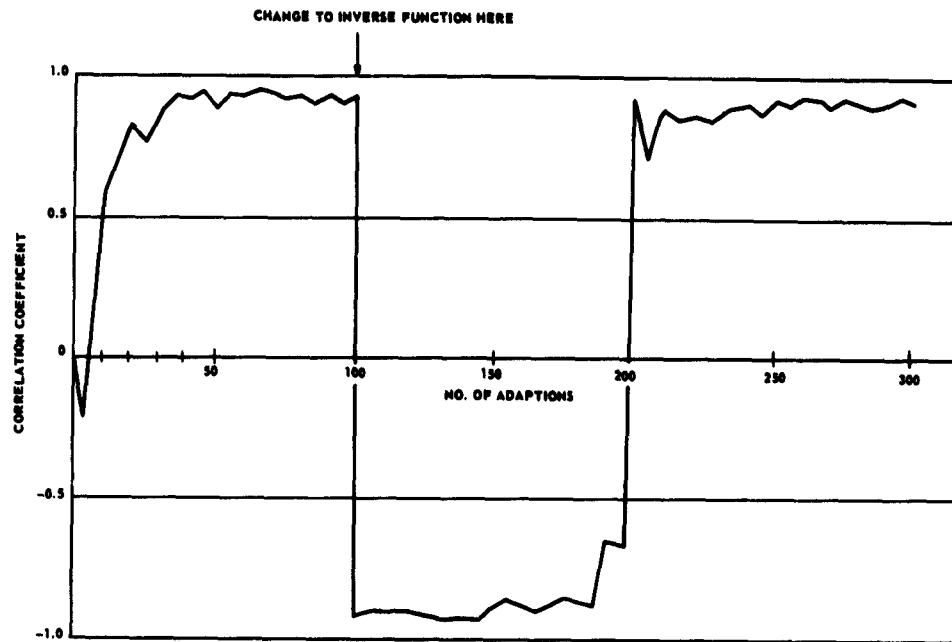
FIG. 10. DYNAMIC BEHAVIOR FOR A CHANGE TO INVERSE PROBABILITY FUNCTION.
 (Parts a through f are on pages 22-24.)



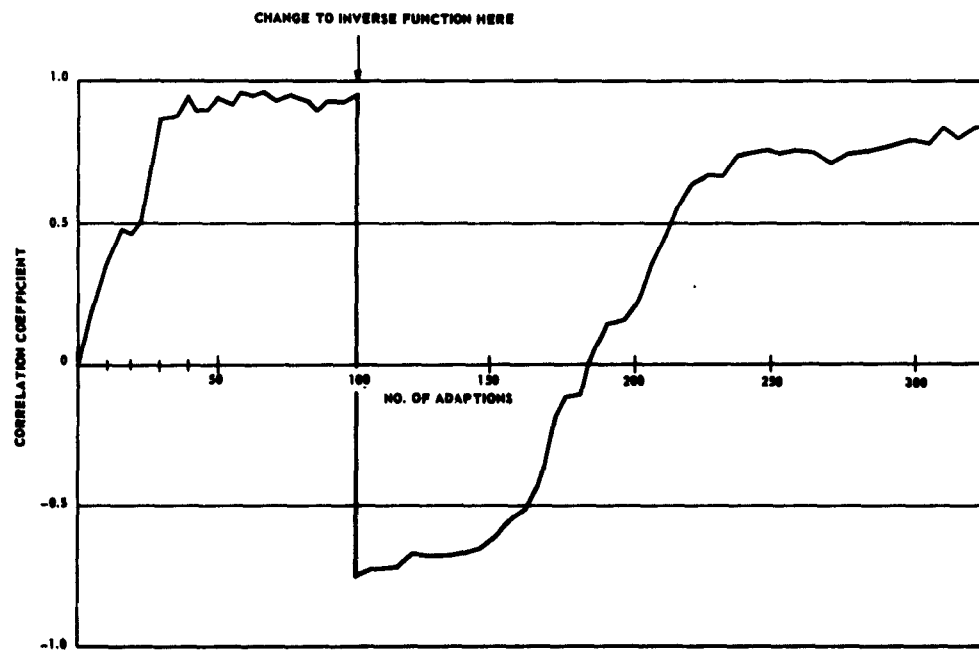
c. 24 random patterns, $C = 6$



d. 10 orthogonal patterns, $C = 12$



e. 10 orthogonal patterns, $C = 6$



f. 24 orthogonal patterns, $C = 6$

The following experiments were conducted over a much larger number of adaptations and the correlation coefficient was evaluated every 100 adaptations only. The patterns were orthogonal. Figure 11 shows behavior for approximately 1000 adaptations before and after a change in input function. First, changes from the same original input function to four different functions are shown and then four consecutive changes after approximately 1000 adaptations are shown. One of the slowest adaptations after the change is the change to the inverse function that was shown in the preceding experiments. Here, however, the adaptation was even slower because it was preceded by 1000 adaptations on the original input function. As would be expected, speed of adaptation is largely influenced by the severity of the change in input function. The fastest recovery occurred after the 3100th adaptation, which involved a fairly mild change in input function.

Figure 12 shows experiments made with the alternate adaptation procedure, in which a fixed correction was made to the confidence level by adding to the gain vector a vector that was proportional to the input-pattern vector. First, a change to the inverse function was made; there was only a small indication of a recovery of the correlation coefficient after 1000 adaptations. Next, a change to an almost flat function showed no trend of recovery. The third change was back to the original input function, which showed good recovery because Adaline was essentially still adapted to that original input.

With this adaptation procedure, the confidence level becomes so large that it is difficult to change it rapidly thereafter. As is evident from the first 1000 adaptations, however, adaptation was exact and complete after 100 adaptations on the original functions (with orthogonal patterns). For a static situation or for very gradually changing dynamic processes, this procedure would be desirable.

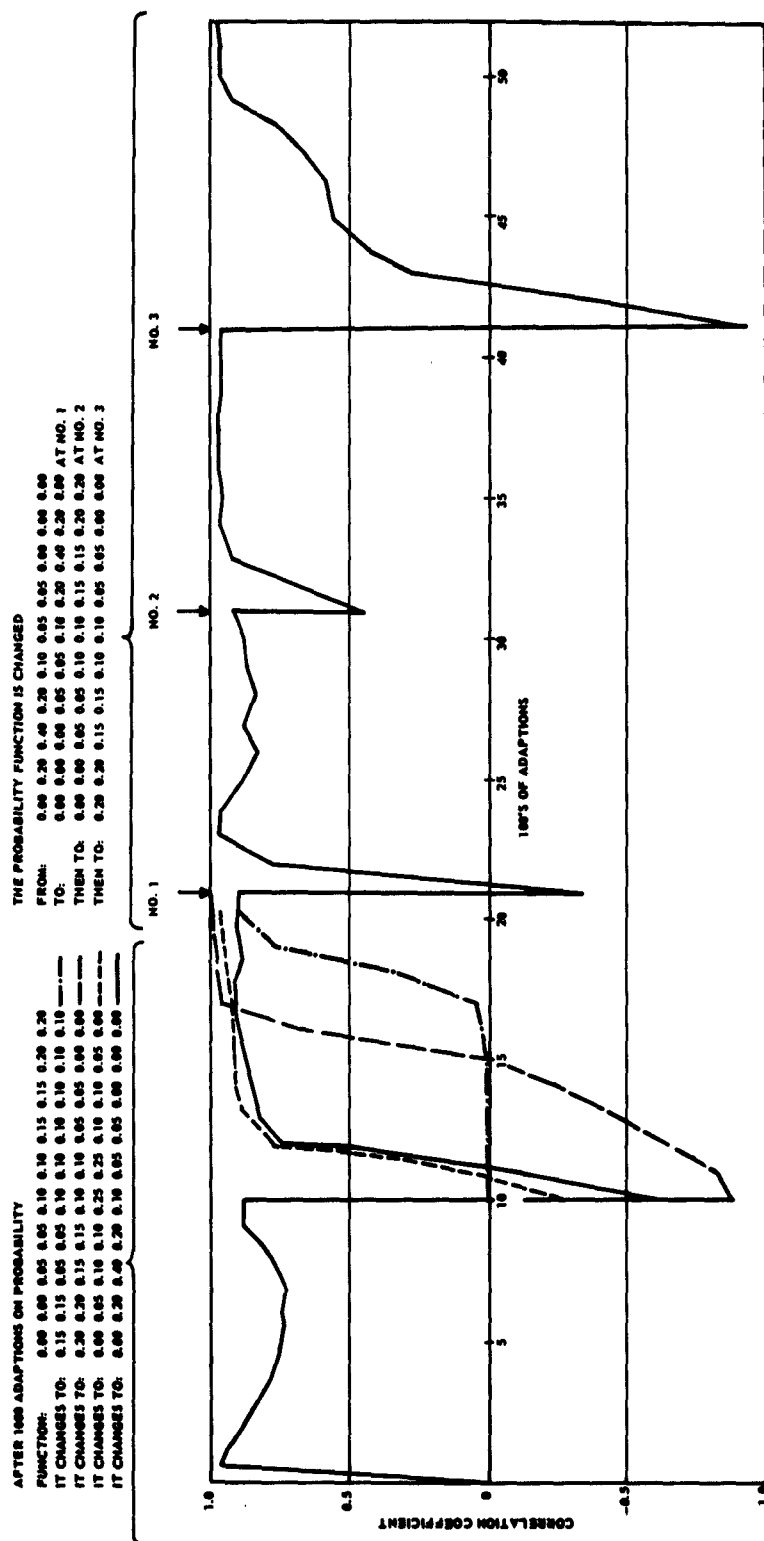


FIG. 11. DYNAMIC BEHAVIOR WITH SUCCESSIVE CHANGES IN PROBABILITY FUNCTION.
 (10 orthogonal patterns, $C = 10$.)

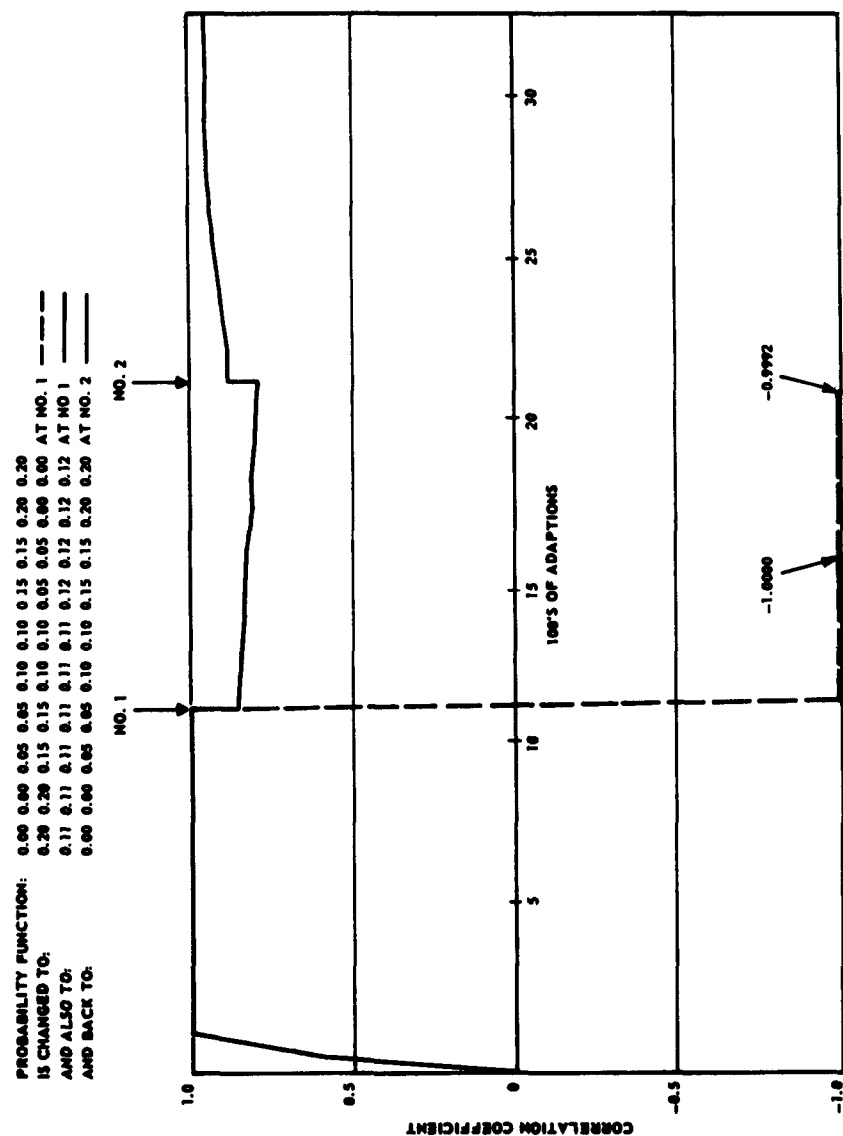


FIG. 12. ADAPTION WITH ALTERNATE PROCEDURE.

VI. FILE SIMULATION

To obtain a better idea of how effective the proposed application of Adaline to a file would be, a complete simulation of a small file was undertaken, including the rearrangement of records in the file and an evaluation of the resultant improvement in average seek time.

Six different input probability functions were constructed. These approximated one-sided normal distributions with 35 to 85 percent of the total activity concentrated in 20 percent of the records. The resulting functions are plotted along the bottom of Fig. 13. These are discrete functions for ten records, which was the total for this file. It was next necessary to have a realistic function representing variation of seek time with record location. There are ten record locations numbered 1 to 10. Seek time (in milliseconds) as a function of record number is shown in Fig. 14, assuming a home position one unit of distance removed from the first record position and all records equidistant. This assumption corresponds to every record in a separate track and seek time proportional to the square root of distance, which is realistic for a bang-bang type of actuator having equal acceleration and deceleration. The actual magnitudes of the seek times serve only to establish a reference for comparison.

With these two assumptions, the best-possible and the worst-possible average seek times were determined. The first would occur when records 1 through 10 happened to be located in locations 1 through 10 respectively. The worst situation would require the reverse order. These seek times were plotted in Fig. 13 vs input probability functions.

Six sets of ten random patterns were next assembled. The file simulation for this case is performed as follows:

1. Pick the first set of ten patterns. Assign these to records 1 through 10. Use the first input probability function.
2. Make 200 adaptations and interrogate after every 100 adaptations to establish two output probability functions.
3. Sort the ten records in order of decreasing probability as given by the output probability function.

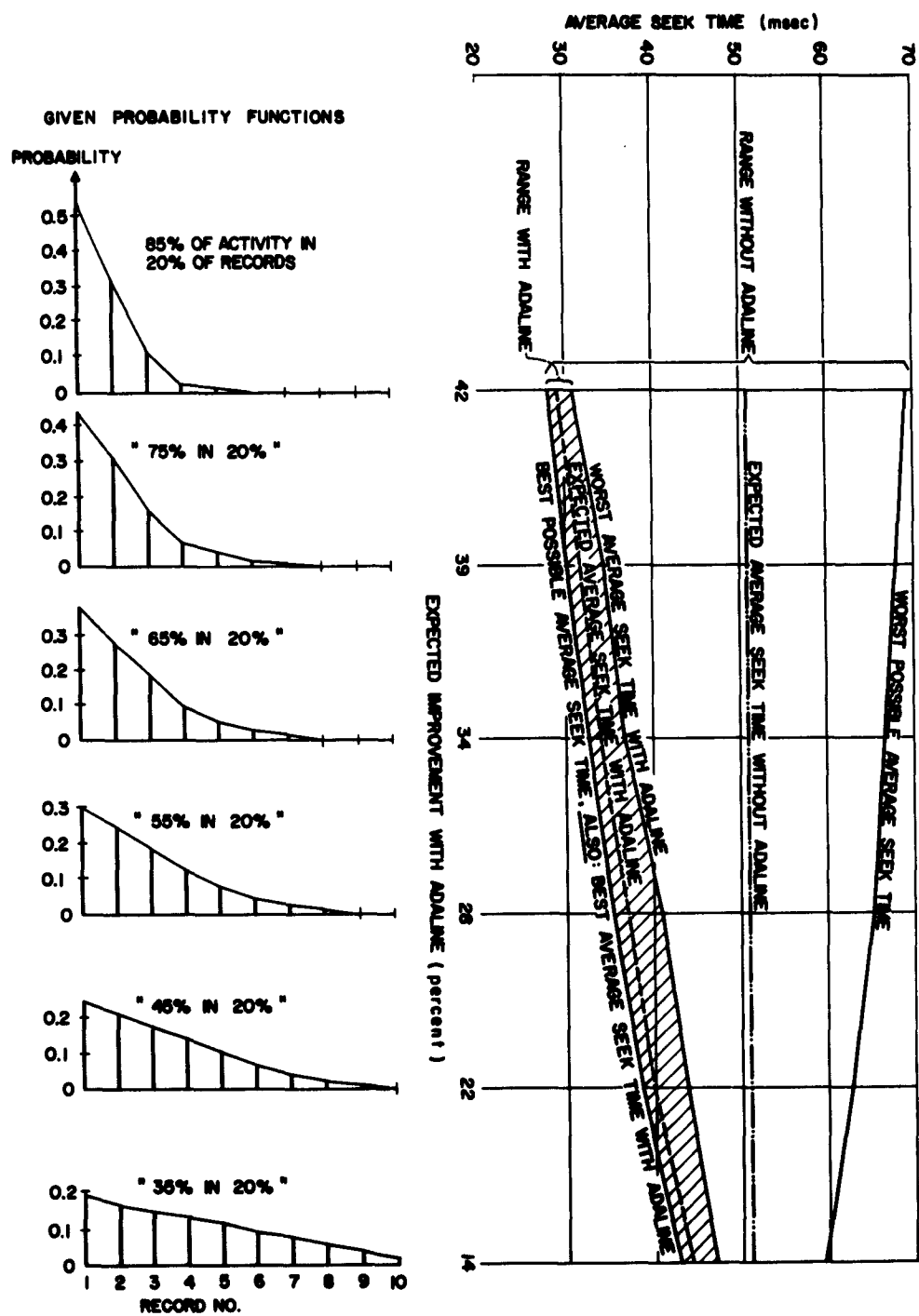


FIG. 13. GRAPHICAL REPRESENTATION OF FILE-SIMULATION DATA.

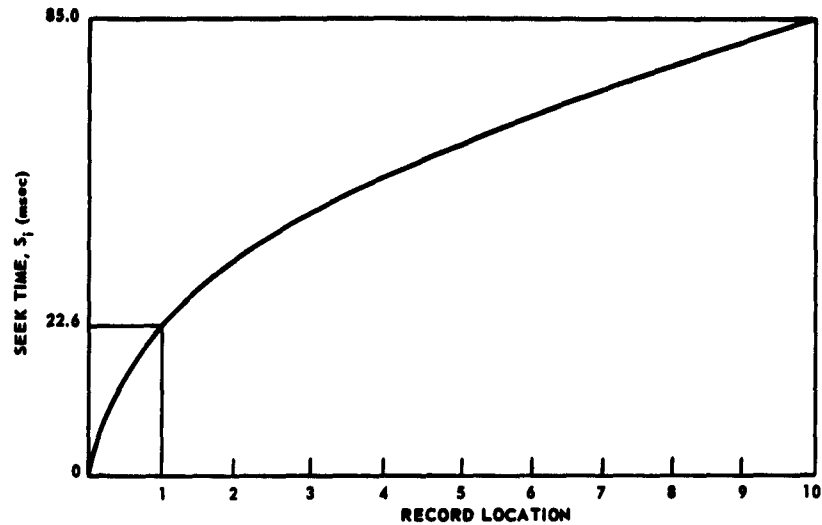


FIG. 14. SEEK TIME FOR THE RANDOM-ACCESS FILE.

4. Compute the average seek time with this new arrangement of records.
5. Step the pattern assignment and repeat items 1 through 4 a total of ten times.
6. Pick the next set of ten patterns and repeat steps 1 through 5 a total of six times.
7. Average the average seek times to get a probable improvement in seek time.
8. Repeat steps 1 through 7 for all six input probability functions.

The results of this simulation are shown in Fig. 13. Plotted vs input function are: the worst average seek times encountered after sorting the records as indicated by the Adaline output after both the first and second 100 adaptions. This plot shows the worst operation using Adaline and compares favorably with the worst operation not using Adaline, which is termed "worst possible average seek time." It also compares favorably with the average or expected operation without Adaline. The best operation with Adaline was plotted which, for all practical purposes,

coincides with the best possible operation in all cases. The average or expected operation with Adaline is also shown and is almost as good as the best possible operation and substantially superior to the expected operation without Adaline. Improvements in expected average seek time with Adaline over operation without Adaline are as high as 42 percent. Of course, where the input function is fairly flat, improvements are small because the possible improvements to be made are limited. The important conclusion to be derived from this simulation of the file as a whole is that the range of average seek times that can be obtained with the use of Adaline is close to the best operation possible. Furthermore, the average of this range is even closer to the best possible operation than the midpoint of the range.

Some clues will be pointed out in Chapter VII which may explain why the operation is statistically this good, even with random patterns.

VII. CROSS EFFECTS IN ADAPTION WITH RANDOM PATTERNS

How can the effect of adaption toward one pattern on the confidence level of another pattern be measured?

Consider the patterns and gains as vectors in an N-dimensional space. The pattern vectors thus end at the corners of an N-dimensional cube and their lengths are equal to the square root of N.

As mentioned in Chapter I, the confidence level is formed as the sum of the products of input pattern elements and the corresponding gains. The confidence level, in vector terms, is then the scalar product of the gain vector and the pattern vector. In the adaption procedure the correction added to the gains is proportional to the error (a scalar) with the sign of the corresponding pattern element. Considered in the N-dimensional space, the correction is a vector parallel to the pattern vector on which adaption is being made. The change in confidence level of another input pattern is, therefore, the scalar product of the correction vector and that input pattern vector; or, the change of the confidence level of another input pattern is proportional to the scalar product of the two pattern vectors. Since the length of the pattern vectors is \sqrt{N} , a good measure of this cross effect would be the scalar product divided by N (which is the cosine of the angle between the two pattern vectors). This can be called the "cross-effect factor."

An estimate of an average value of this factor for any dimension can be obtained by first looking qualitatively at the two- and three-dimensional space. In the two-dimensional case, there are only three pattern vectors in addition to the one under adaption. One pattern vector is the equal and opposite of the latter, which will be the case in any dimension; the other two are orthogonal to it. In other words, two of the three additional pattern vectors are orthogonal to the pattern vector being adapted on. Now, in three-dimensional space, all the pattern vectors end in the corners of a cube. Consider the cube tilted so that the diagonal that contains the adapted-on vector becomes vertical. It then appears that all the pattern vectors not on that diagonal are "not too far" from a horizontal plane through the center

of the cube. This observation is, perhaps, an indication that the configuration will turn out similarly in higher dimensional cases and the average scalar product will be much less than N.

Now look at the probable magnitude of this scalar product in N-dimensional space. There are $(N + 1)$ possible values of this scalar product, including the product with itself, which is included for symmetry and because the final form of the average magnitude assumes a neater form. The actual value of the average magnitude is barely affected by this inclusion for higher dimensions. The $(N + 1)$ values represent the case where: (1) all the elements are the same, (2) one element is different, (3) two are different, and so on down to all elements being different. Let i denote the number of elements that are different; then i ranges from 0 to N. Let

p = probability that any two elements being multiplied in the scalar product are the same = 0.5

q = probability that they are different = 0.5

Let the pattern vectors be

$$A(a_1, a_2, a_3, a_4, a_5, \dots, a_N)$$

and

$$Y(y_1, y_2, y_3, y_4, y_5, \dots, y_N)$$

Then the scalar product

$$A \cdot Y = a_1 y_1 + a_2 y_2 + a_3 y_3 + \dots + a_N y_N$$

And for any i the scalar products and their probabilities are tabulated as follows:

i	A · Y	Probability
0	N	$p^N = 2^{-N}$
1	N - 2	$N \cdot p^{N-1} \cdot q = N \cdot 2^{-N}$
2	N - 4	$N \cdot (N - 1)/2 \cdot 2^{-N}$
3	N - 6	$\binom{N}{3} \cdot 2^{-N}$
.	.	.
.	.	.
.	.	.
1	N - 2i	$\binom{N}{1} \cdot 2^{-N}$
.	.	.
.	.	.
.	.	.
$\frac{N}{2}$ (if N = even)	0	$\binom{N}{N/2} \cdot 2^{-N}$
.	.	.
.	.	.
.	.	.
N	-N	$\binom{N}{N} \cdot 2^{-N}$

Or, dividing the magnitude of the scalar product by N gives the

probability of $(N - 2i)/N$ occurring as being $\binom{N}{i} \cdot 2^{-N}$.

This expression is plotted for $N = 2$ and $N = 5$ in Fig. 15, and for $N = 32$ in Fig. 16. The fact that the mean is equal to 0 assures that, if Adaline adapts on a large number of patterns, the effect on some other pattern is going to be small. If Adaline adapts many times on only a few or on only one pattern, what would the effect be on another pattern? If Adaline adapts to a desired output of +1 or -1 and makes many adaptations

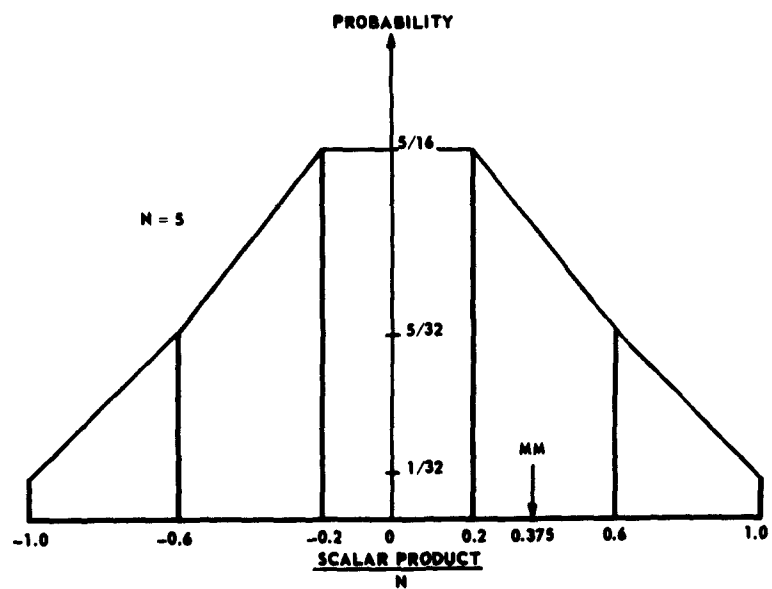
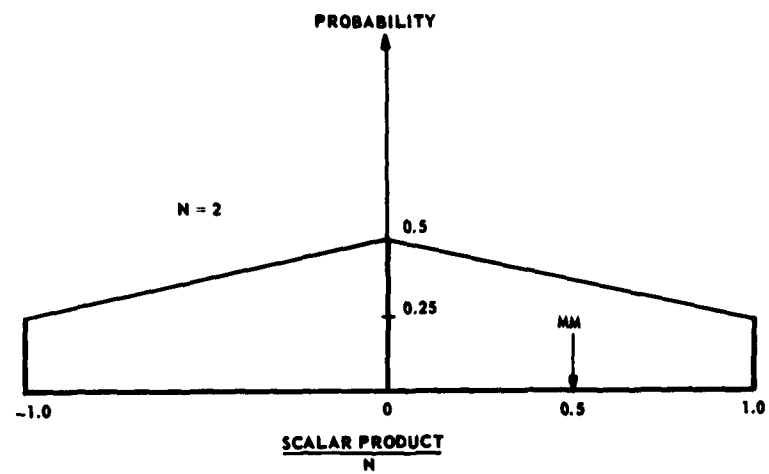


FIG. 15. PROBABILITY FUNCTION FOR CROSS-EFFECT COEFFICIENT FOR $N = 2$ AND $N = 5$.

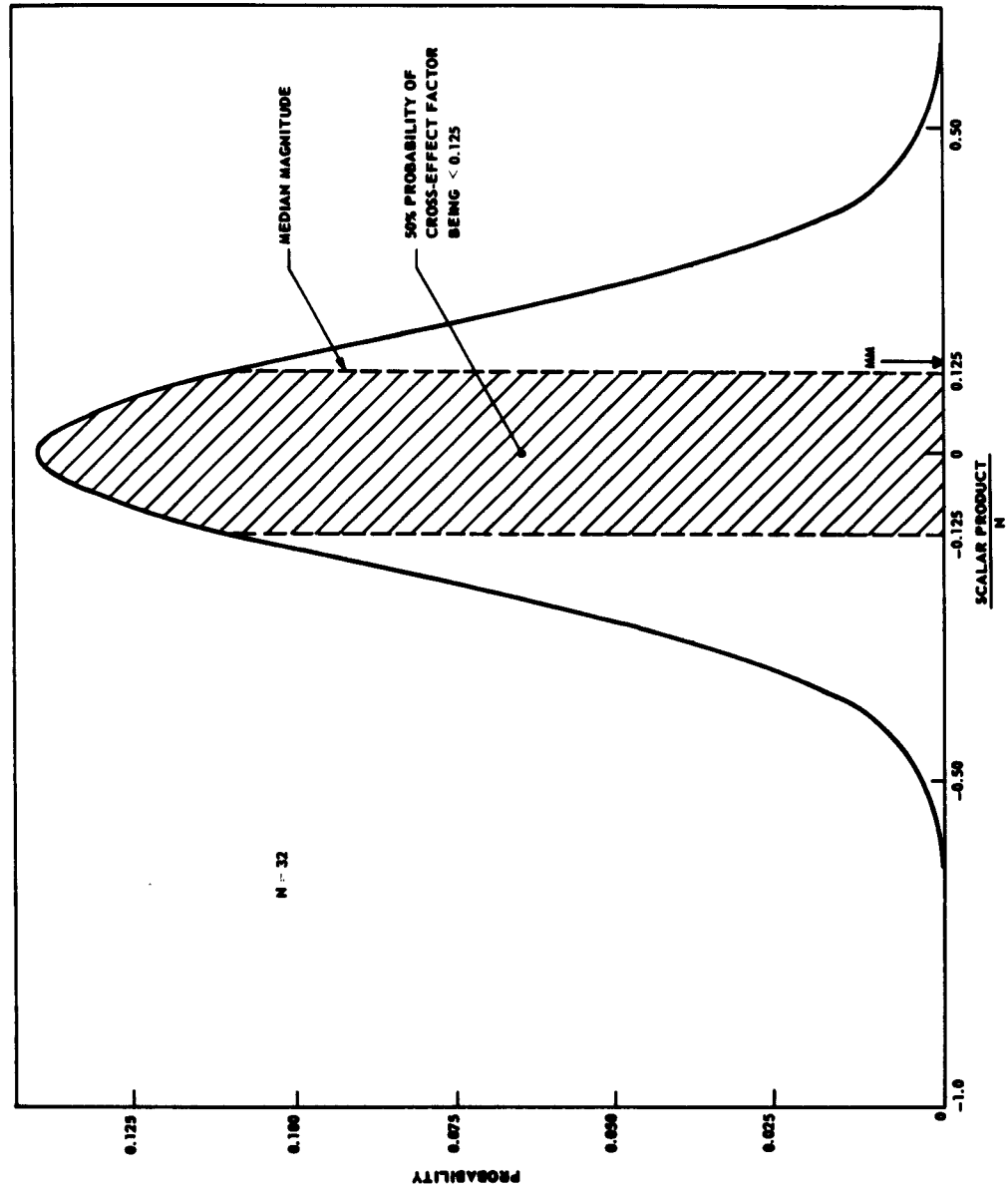


FIG. 16. PROBABILITY FUNCTION FOR CROSS-EFFECT COEFFICIENT FOR $N = 32$.

on the same input pattern with a time-constant fact $C > 1$, the correction vector will decrease in magnitude and the cross effect will diminish accordingly. It is still desired to know a measure of the likelihood of the cross-effect factor approaching 1. The mean of the absolute value of the cross-effect factor or the median of the absolute value will shed some light on that question. First, consider the mean of the absolute value or mean magnitude (MM) or first moment of the absolute value of the probability function. The mean magnitude is indicated in Figs. 15 and 16.

$$\begin{aligned} \text{MM} &= \text{sum of} \left(\text{probabilities} \times \frac{|N - 2i|}{N} \right) \\ &= \frac{1}{N} \cdot \left[2^{-N} \sum_{i=0}^N \binom{N}{i} \cdot |N - 2i| \right] \end{aligned}$$

If $N = \text{even}$,

$$\text{MM} = \text{MM}_E$$

$$\begin{aligned} \text{MM}_E &= \frac{1}{N} \cdot 2^{-N} \cdot 2 \sum_{i=0}^{(N/2)-1} \binom{N}{i} \cdot (N - 2i) \\ &= \frac{2^{-N}}{N} \cdot \left[4 \sum_{i=0}^{(N/2)-1} \binom{N}{i} (N - i) - 2N \sum_{i=0}^{(N/2)-1} \binom{N}{i} \right] \end{aligned}$$

where

$$\binom{N}{i} (N - i) = \frac{N! (N - i)}{(N - i)! i!} = \frac{(N - 1)! N}{(N - 1 - i)! i!} = N \binom{N-1}{i}$$

$$MM_E = \frac{2^{-N} \cdot N}{N} \left[4 \sum_{i=0}^{(N/2)-1} \binom{N-1}{i} - 2 \sum_{i=0}^{(N/2)-1} \binom{N}{i} \right]$$

Now

$$\sum_{i=0}^{N-1} \binom{N-1}{i} = 2^{N-1}$$

has N terms if $N = \text{even}$ and

$$\sum_{i=0}^{(N/2)-1} \binom{N-1}{i} = \frac{1}{2} \cdot 2^{N-1} = 2^{N-2}$$

since it contains $N/2$ terms of the above expression and

$$2 \sum_{i=0}^{(N/2)-1} \binom{N}{i} = \sum_{i=0}^N \binom{N}{i} - \binom{N}{N/2} = 2^N - \binom{N}{N/2}$$

because the left-hand side equals twice the first $N/2$ terms of

$$\begin{aligned} & 2^N - \binom{N}{N/2} \\ MM_E &= 2^{-N} \cdot \left\{ \cancel{2^N} - \left[\cancel{2^N} - \binom{N}{N/2} \right] \right\} \\ &= 2^{-N} \cdot \binom{N}{N/2} = \frac{N}{N+1} \cdot \frac{N!}{\left[\left(\frac{N}{2} \right)! \right]^2} \end{aligned}$$

which by Stirling's formula:

$$\approx \frac{1}{\sqrt{2\pi(N/2)}} \cdot \frac{(2N\pi)^{1/2}}{2 \cdot (N\pi)} \approx \sqrt{\frac{2}{N\pi}}$$

For $N = 32$,

$$MM_E = 0.141 \quad \text{Quite small!}$$

If $N = \text{odd}$,

$$\begin{aligned} MM = MM_0 &= \frac{1}{N} \cdot \left[2^{-N} \sum_{i=0}^N \binom{N}{i} \cdot |N - 2i| \right] \\ &= \frac{1}{N} \cdot 2^{-N} \left[\sum_{i=0}^{(N-1)/2} \binom{N}{i} \cdot (N - 2i) - \sum_{i=(N+1)/2}^N \binom{N}{i} (N - 2i) \right] \end{aligned}$$

Let $i = N - j$

$$MM_0 = \frac{2^{-N}}{N} \left[\sum_{i=0}^{(N-1)/2} \binom{N}{i} (N - 2i) + \sum_{j=0}^{(N-1)/2} \underbrace{\binom{N}{N-j}}_{\binom{N}{j}} (N - 2j) \right]$$

$$\text{where } \binom{N}{N-j} = \binom{N}{j}$$

$$= \frac{2^{-N}}{N} \left[2 \cdot \sum_{i=0}^{(N-1)/2} \binom{N}{i} \cdot (N - 2i) \right]$$

$$MM_0 = \frac{2^{-N}}{N} \left[4 \sum_{i=0}^{(N-1)/2} \binom{N}{i} \cdot (N-1) - 2 \sum_{i=0}^{(N-1)/2} \binom{N}{i} \right]$$

$$\text{where } \binom{N}{i} \cdot (N-1) = N \binom{N-1}{i}$$

$$= \frac{2^{-N}}{N} \cdot N \left[4 \sum_{i=0}^{(N-1)/2} \binom{N-1}{i} - 2 \sum_{i=0}^{(N-1)/2} \binom{N-1}{i} \right]$$

$$= 2^{-N} \left[4 \cdot \frac{1}{2} \left\{ \cancel{2^{N-1}} + \binom{N-1}{(N-1)/2} \right\} - 2 \cdot \left(\frac{1}{2} \cdot 2^N \right) \right]$$

$$= 2^{-N} \cdot 2 \binom{N-1}{(N-1)/2} = 2^{-(N-1)} \binom{N-1}{(N-1)/2}$$

which is equal to the next lower MM_E . (This result is exact, before applying Stirling's formula.) In other words,

$$MM_0(N) = MM_E(N-1)$$

This mean magnitude was plotted vs N in Fig. 17 for both even and odd N 's. The median of the magnitude was also computed and plotted in the same figure. Generally the median is even smaller than the mean, which is true for $N = 32$ as shown in Fig. 16. In this case we have a probability of 0.5 that the effect of adaption on one pattern is going to affect another confidence level by less than 12.5 percent of the correction made.

It is realized that the above discussion has omitted the effect of the "bootstrap" mode of picking the desired output. It is not immediately

obvious whether or not that mode would reduce the cross-effect mean even further.

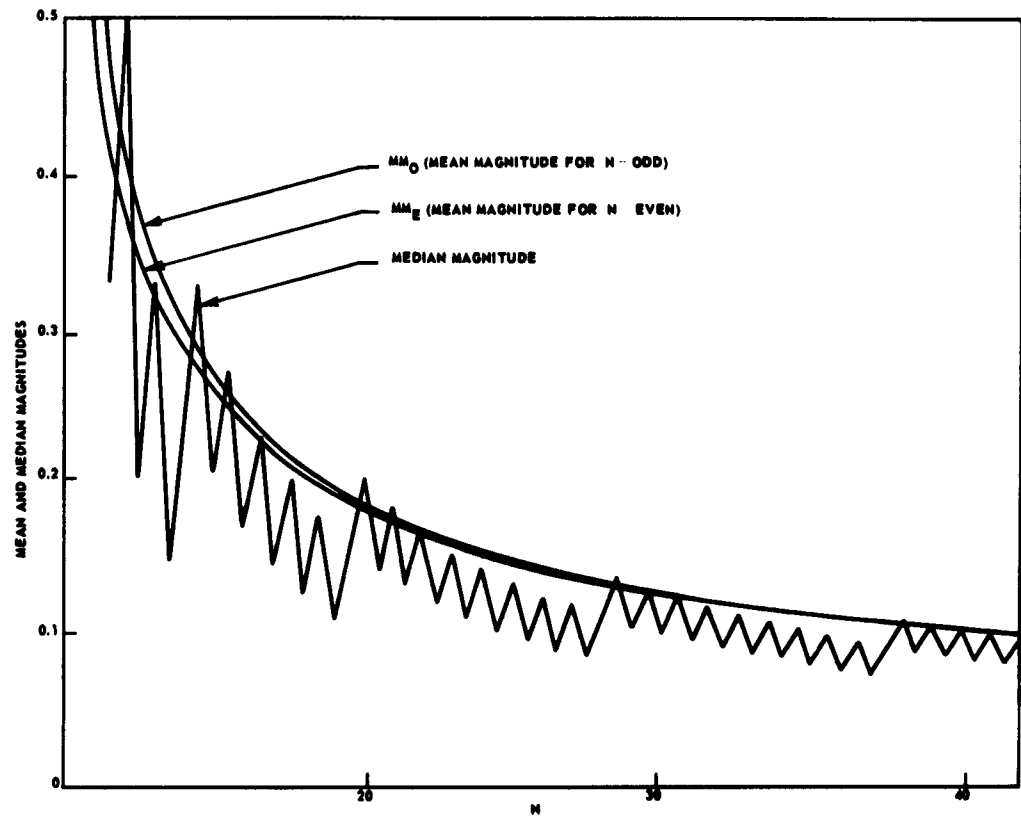


FIG. 17. MEAN MAGNITUDE OF CROSS-EFFECT COEFFICIENT.

VIII. CONCLUSION

The small size of the mean and median magnitudes of the cross-effect factor explains to some extent the successful operation with the use of Adaline in the file-simulation experiment, even with random patterns. It should therefore be possible, with some simple algorithm, to construct the patterns assigned to a record from the identification portion of the record or from the record information directly. For best operation this algorithm should result in a unique pattern being assigned to each record, but this is not absolutely necessary.

The above results may be applied to a more general problem, where it is desired to get a comparative measure of the frequency of occurrence of a number of events as in Fig. 1. The file simulation has demonstrated the feasibility of using Adaline by assigning random patterns to each event that is to be counted, and proceeding as indicated in Fig. 1.

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